

Wilfried Neumaier, *Was ist ein Tonsystem? Eine historisch-systematische Theorie der abendländischen Tonsysteme, gegründet auf die antiken Theoretiker Aristoxenos, Eukleides und Ptolemaios, dargestellt mit Mitteln der modernen Algebra.* (Quellen und Studien zur Musikgeschichte von der Antike bis in die Gegenwart, Band 9). Frankfurt a.M. etc.: Peter Lang, 1986.

Published in *Historia Mathematica* 17 (1990), 172–174.

Neumaier's book arose as a dissertation at Tübingen University; this origin is mirrored in many of the features of the work, not least its solid methodological and philosophical reflections. The author's aim is to offer new insights into the nature of tonal systems through the integration of historical schemes into an axiomatic system, formulated in a language inspired by contemporary abstract algebra but organized according to the classical Aristotelian ideal that concepts, definitions and axioms for a particular field of knowledge should belong specifically to that field – in the present case they should thus build on auditive experience. Neumaier's particular integration of systematic and historical structures could only have been produced by an Aristotelian temperament who has passed through Hegelian philosophy and been trained in formalist mathematics.

The book consists of three "parts", of which the initial part A builds up the concepts necessary to construct an "auditive tonal system", a concept which is distinguished both from "acoustical systems" based on the physics of sound and from mathematical models based on proportions. Here and throughout the book, the strict group-theoretical formulation is relegated to footnotes.

Part B takes up the historical approach, concentrating on two paradigmatic schemes from Greek antiquity, the "auditive" system of the Peripatetic Aristoxenos and the Pythagorean tradition based on ratios as reformulated in Euclid's *Sectio canonis*. Neumaier, who is evidently congenial with Aristoxenos and more sympathetic to his approach than most historians of mathematics[1], analyzes both systems and expounds their incompatibility on all levels from epistemological principles to musical subject-matter. In this connection he shows that while Aristoxenos' axioms are fulfilled in an equal temperament, the idea of temperament is irrelevant for the understanding of Aristoxenos, who simply chooses to neglect the very existence of the problems addressed by it.

As a spin-off from the main analysis, Neumaier proposes a reconstruction of book III of Aristoxenos' *Harmonic Elements* based on strong and clear arguments for what must be original and what must be secondary insertions. A similar analysis of the *Sectio canonis* shows that its second part should be understood as a non-polemical response to Aristoxenos. For this reason and because of the inner coherence of the work and its relation to the *Elements*, Neumaier concludes that the *Sectio* is correctly ascribed to Euclid and is neither a compilation from disparate sources nor a collection of excerpts

from some larger Euclidean treatise.

The final chapter of part *B* deals with the relation of Ptolemy's *Harmonics* to the previous two authors' works (Ptolemy's position is claimed, by arguments which in the reviewer's opinion are sound but not compelling, to go back to Eratosthenes). Ptolemy is shown to have known Aristoxenos only indirectly through "Aristoxeneans" who did not understand their master. This is one of the reasons Ptolemy ends up interpreting Aristoxenos, whose project is not far from his own pattern of thought, precisely from that Pythagorean point of view which he explicitly characterizes as too speculative. Another reason becomes clear from Ptolemy's criticism of Aristoxenos' linear measurement of intervals (cited on p. 151), which can only arise as differences between numbers ascribed to the terminal tones. Only after the concept of logarithms had been developed could such numbers, which ascribe an arithmetical series e.g. to a sequence of octaves, be compatible with the (usual) ascription of a geometrical series to the same sequence, which ultimately refers to physical experiments and forms the basis of Pythagorean harmonics. Ptolemy's synthesis of the two approaches thus *had* to be a theoretically shaky compromise, given the mathematical apparatus available in antiquity.

After the discussion of Ptolemy, a brief introduction to Boethius' tonal system is given. This opens the way to part *C*, where Neumaier attempts a synthesis of the Aristoxenean and Euclidean approaches on contemporary foundations. He continues the historical approach, following the development of the Boethian system from Odo of Cluny and Guido of Arezzo, who introduced modern tonal terminology around the turn of the millennium, until the development of the mean-tone and equal temperament and the first descriptions based on roots and logarithms in the sixteenth and seventeenth centuries. He demonstrates that even the *musica theorica* of the Middle Ages was transformed under the pressure of musical practice. Other strands of Neumaier's argument in part *C* include a discussion of the principles which must be followed if synthesis is to arise; the development of a framework into which all historical positions can be fitted; and sketches of the actual fitting.

It will come as no surprise that this synthesis turns out to be an idealized auditive system isomorphic to the infinite scale of frequencies. As always when a synthesis between apparently incompatible viewpoints is formulated mathematically, the offspring engendered by the dialectical mountain is reduced by the powerful means of mathematical abstraction to an apparent mouse. What is gained from the integration of historical material is increased awareness of the range and possibilities of a system which in itself is not very new, Aristotelian ideals apart.

And as always when a synthesis is formulated as abstract mathematics there are also losses. A loss which seems to have escaped Neumaier is that Aristoxenos' system has to be (and is in fact) (mis)interpreted as equally tempered in order to fit. Other losses are suffered by the formal axiomatic theory (which will probably be less regretted by both historians and musicians). Even though tonal spaces (of which tonal systems

are understood as subsets) are defined as "sets" of undefined "tones", and intervals as "ordered sets of two tones", the tones and intervals of *different* tonal spaces are presumed to be *the same* (and hence the real physical or auditive) tones and intervals. Similarly, the "octav" of the abstract system, which could in fact correspond to any interval (or changing intervals) in the physical application of the theory, is tacitly understood as a normal octave. Though dressed up as purely formal axiomatics, Neumaier's system turns out to be even more Aristotelian in character than intended.

If we turn from the general point of view to that of the historian, the descriptions of Aristoxenos, Euclid, and Ptolemy yield valuable new insights. As a general treatment of ancient harmonics, however, part *B* is flawed (and no wonder, since the intention of the work is different). In particular, the connections between harmonics and other mathematical disciplines are not exploited even when they seem obvious. Yet at least the arithmetical theories and terminologies of *means* and *proportions* have much to offer (as has been pointed out the other way round by Arpád Szabó). Nonetheless, the historian interested in harmonics should take note of the work.

The book is provided with indices of mathematical concepts, of musical concepts, and of names, with an explanatory list of mathematical concepts (ranging from those of mathematical logic to those describing ordered groups). The latter contains a few errors (worst is a corrupt description of the "inverse function"), but probably nothing which will impede the understanding of readers able to follow the symbolism. In a scheme of intervals taken from Boethius, "2196" should be changed everywhere to "2916" (p. 159, three times).

Jens Høyrup

NOTE

1. We may quote Winnington-Ingram's standpoint [1970, 282]: "It is paradoxical [that Aristoxenos should be included in a dictionary of scientific biography] because he turned his back upon the mathematical knowledge of his time to adopt and propagate a radically 'unscientific' approach ...".

REFERENCES

Winnington-Ingram, R. P., 1970. "Aristoxenos". *Dictionary of Scientific Biography* I, 281-283. New York: Scribner.